7. Approaches To Public Utility Pricing

1. Introduction

This chapter is an attempt to review some important problems in the theory of public utility pricing with particular emphasis on postal services. Some of these problems have their genesis in a distinguishing characteristic of many public utilities : joint production of several items with increasing returns to scale. This poses the difficult problem of allocation of joint costs among these items.

There are two interrelated but analytically separable issues here:

(i) The fact that there are increasing returns to scale may lead to the development of the 'natural' monopoly.

(ii) Because several items are being jointly supplied, each such item must have adequate incentive to stay with the 'monopoly' or, alternatively, be able to face the competition from another agency that may be supplying just this item or a subset of such items. Any regulation of this 'monopoly' must pay heed to these issues.

One method of regulation that is able to give due emphasis to both these issues and which has been the subject of intense scrutiny in the literature is the so-called cross-subsidy-free prices which are Pareto aptimal. Another such regulatory mechanism is second best Ramsey pricing. In the following sections of this chapter we discuss some of the these pricing schemes for supplying public utility services.

The plan of the remaining part of this chapter is as follows. In Section 2 we consider the problem of determining the first best prices for a public utility that produces many goods jointly. In particular, we examine the proposition that subsidy-free prices are Pareto optimal. In Section 3 we study Aumann-Shapley prices which, it is claimed, are subsidy-free and have certain other desirable properties as well. Section 4 considers the problem of second best pricing of public utilities in the sense that the problem of cross-subsidisation is ignored and the public utility maximises a welfare function subject to a pre-specified budget constraint. Section 5 presents a case in which we have Aumann-Shapley prices for the public utility and welfare maximising subsidy/tax-inclusive prices for final consumers of utility services and the final section provides conclusions.

2. Subsidy-Free Prices

An important characteristic of many public utilities is that they provide many goods/services simultaneously and the cost of producing these goods/services are not completely allocable among them. The genesis of the problem lies in the fact that there are economies of scale in joint production which are left unexploited, if each item is produced by a different producer. The overall cost of provision of postal services would be higher if first class mail services like post cards were produced by one agency and parcel service was provided by another and so on. Suppose the postal department provides N services, levels of which are denoted by $\bar{q}_1, \ldots, \bar{q}_N$. If $C(\bar{q}_1, \ldots, \bar{q}_N)$ is the cost of providing these services, then it follows that this cost would be lower than the cost that could be attained by the arrangement that permits the provision of these services by more than one agency, i.e., for every subset S of these N services it must be true that

$$C(\bar{q}_1, \ldots, \bar{q}_N) \leq C_s(\bar{q}_1, \ldots, \bar{q}_S) + C_{n-s}(\bar{q}_{s+1}, \ldots, \bar{q}_N) \quad (1)$$

where C_s (.) is the cost of providing S services at the required levels $q_1, \ldots, q_S$ and , correspondingly, C_{N-s} (.) is the cost of providing the other services at the pre-specified levels.

In a market with free entry and free exit, inequality (1) implies that it is profitable to supply all services by one firm. But in an unregulated situation, the firm may become a 'natural' monopoly charging prices which are Pareto inefficient. The problem then is to look for a pricing scheme to regulate the firm for yielding Pareto efficient outcomes. Faulhaber (1975) has derived in a pioneering paper one such set of pricing rules using the theory of cooperative games. If we designate each service as a player in a cooperative game, given the inequality (1), it is profitable to have a grand coalition rather than forming smaller and less profitable sub-coalitions of players and services. Then the problem is to find out the prices which will induce and preserve the grand coalition, i.e., the single supplier arrangement. For example, in the case of Indian postal services, although the various services of the Post Office are technically not free to form smaller sub-coalitions, there is still the problem that it might be profitable for other (private) agencies to provide some of these services. In either case, in the absence of right structure of prices, the grand coalition is potentially unstable.

Suppose that such a set of prices which induces each constituent service to stay in the grand coalition has been found. What must be its characteristics? These must relate to (for any constituent services) profitability inside and outside the grand coalition. Let $P_1, \ldots, p_N$ be the prices for these N services that induce all of them to stay in the grand coalition. Assuming independent demands, the demand for service i at price p_i is q_i (p_i) for i = 1, 2 . . . , N. Then it must be true that

$$\sum_{i=1}^{N} p_i q_i - C(q_1, \ldots, q_N) = \pi(.) = 0 \quad (2)$$

$$\text{and that} \quad \sum_{i=1}^{S} p_i q_i \leq C(q_1, \ldots, q_s) \tag{3}$$

for any subset S of N services. By substracting (3) from (2) we have

$$\sum_{i=1}^{S} p_i q_i \geq C(q_i, \ldots, q_N) - C(q_{s+1}, \ldots, q_N) \tag{4}$$

i.e., the revenue contributed by the set of services $S \leq N$ should be at least as great as the added cost of supplying S. The prices satisfying the inequality (4) are called subsidy-free prices.

"If the provision of any commodity (or group of commodities) by multi-commodity enterprise subject to a profit constraint leads to prices for the other commodities no higher than they would pay by themselves, then the price structure is subsidy-free. Thus, a subsidy-free price structure insures that the provision of each commodity by the enterprise is 'Pareto Superior' to non-provision" (Faulhaber, 1975). Since a set of subsidy-free prices defined above induces rational players in the game to cooperate and is stable against all possible coalitions, it belongs to the core of the game. An immediate corollary is that this solution is also Pareto optimal so far as these N services are concerned.

If we did not have cross-subsidy-free prices, some services would be subsidising others and, since they could do better in smaller coalitions, there would be incentives for the former to leave the grand coalition. Also, there could be incentives for some new agencies to supply those services that are subsidising others in the grand coalition. If cross-subsidisation does exist, then it follows that these agencies could profitably compete. Cross-subsidy-free prices are efficient prices. They also ensure that the grand coalition breaks even.

3. Computation of Subsidy-Free Prices

If costs of production in a multi-product public utility were fully allocable among the various services, then cross-subsidy-free prices would require each service to have a price that just covers its own cost. Figure 1 represents the long-run cost structure of one of the services offered by a multi-product firm. Then the question is, what is the price for the services that cover the full cost at any given level of supply. This price may be defined as

$$p_i = \int_o^1 \frac{\delta C(tq_1, tq_2, \ldots, tq_N)}{\delta q_i} \, dt \quad i = 1, 2, \ldots, N \qquad (5)$$

where $C_i\,(q_1, q_2 \ldots q_N)$ is the long-run cost function with $0 \leq t \leq 1$. At the level of production q_i^* the total cost of q_i is given as

$$C_i\,(q_i^*) = \int_o q_i^* = \int_o q_i^* \frac{\delta C_i}{\delta q_i} \, dq_i$$

which is equivalent to the shaded area in Figure 1. If p_i ($i = 1, 2, \ldots, N$) are full cost prices, we have

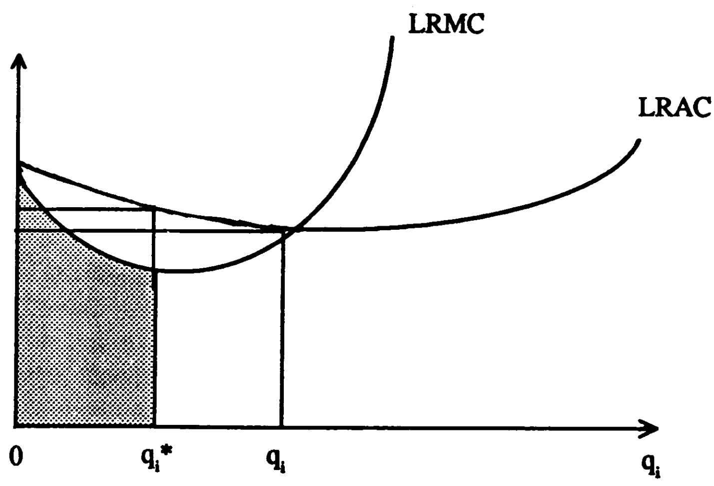

Figure - 1

$$\sum_{i=1}^{N} p_i (q_i^*)\, q_i^* = \sum_{i=1}^{N} \int_{0}^{q_i^*} \frac{C_i}{q_i}\, dq_i \qquad (6)$$

The prices defined in (5) are known as Aumann-Shapley prices (A-S prices).[1] For a cost-minimising firm with decreasing or constant returns to scale, A-S prices and marginal cost prices are the same. At cost-minimising quantity q_i, the price that is equal to marginal cost is also the A-S price that covers full cost, as can be seen in Figure 1. Also, for a single-product firm, the price defined in (5) is same thing as average cost. However, in the case of a multi-product firm with joint variable cost in the long run, the sharing of joint costs by different commodities is implicit in A-S prices which are free from cross-subsidisation. These prices may be used by regulated monopolies and public and quasi-public agencies to allocate the joint cost of production to different commodities produced by them. To compute A-S prices, only the cost structure and output vector must be known.

Billera and Heath (1982) and Mirman and Tauman (1982) have proposed an axiomatic approach to cross-subsidy-free or A-S prices for multi-product public utility. They have shown that A-S price mechanism is the only price mechanism which sastisfies the following five axioms for continuously differentiable cost functions with no fixed cost components.

1. *Cost Sharing :* The prices of various commodities of a multi-product firm are such that they cover the full cost of production

$$\sum_{i=1}^{N} p_i\, q_i = C\,(q_1, \ldots, q_N)$$

i.e., the total cost is equal to total revenue.

2. *Rescaling :* If the scales of measurement of commodities are

1. See R. J. Aumann and L.S. Shapley (1974) and L.J. Billera, D.C. Heath and J. Raanan (1979).

changed, then the prices are changed accordingly. If the cost functions of a multi-product firm differ only with respect to scales of commodities

$$G(X_1, \ldots, X_N) = C(q_1, \ldots, q_N)$$

then

$$P(X_i) = \lambda p(q_i), \; i = 1, 2, \ldots, N.$$

3. *Consistency* : Each unit of the same good has the same price. If prices depend only upon cost functions and not demand functions, being the same good will mean, the good with same cost.

$$\text{Let } C(q_1, \ldots, q_N) = G\left(\sum_{i=1}^{N} q_i\right)$$

then

$$p_i = p\left(\sum_{i=1}^{N} q_i\right), \; i = 1, 2, \ldots, N.$$

4. *Positivity* : If a cost function C increases at least as rapidly as the cost function G with respect to quantities of commodities, then the prices determined for C should be at least as high as those determined for G.

$$p_i(C) \geq p_i(G^*)$$

$$\text{if } \frac{\delta C}{\delta q_i} \geq \frac{\delta G^*}{\delta q_i}, \; i = 1, 2, \ldots, N$$

5. *Additivity* : If a cost function can be broken down into two components say C and G* (e.g., management and production), then calculating the price determined by the cost function for any level of production can be done by adding the price determined by C and G* respectively for that level of production. That means

$$p_i\,(C+G^*) = p_i\,(C) + p_i\,(G^*) \qquad i = 1, \ldots, N$$

A-S prices that satisfy these axioms can be computed given the long-run cost functions with no fixed-cost elements. However, the short-run cost functions have fixed-cost components which are normally joint costs for a multi-product firm. In this case allocation of fixed costs among different commodities may be possible, given the information about both long-run and short-run cost functions. Given the envelope theorem of long-run cost functions, the efficient portion of short-run technology coincides with long-run technology used by the firm. Using A-S prices for the long-run cost function, an allocation of fixed cost associated with the efficient (short-run) technology can be determined. Suppose the cost function is given by

$$G\,(q_1, \ldots, q_N) = C\,(q_1, \ldots, q_N) + F \tag{7}$$

where C and F are respectively variable and fixed costs. A-S prices with long-run cost functions are given by

$$p_i = \int_0^1 \frac{\delta G(tq_1, \ldots, tq_N)}{dq_i}\, dt, \; i = 1, 2, \ldots, N \tag{8}$$

which cover both variable cost and fixed cost. We can then compute another set of prices p_i, $i = 1, 2 \ldots N$ which cover only variable cost as

$$p_i = \int_0^1 \frac{\delta C(tq_1, \ldots, tq_N)}{dq_i}\, dt, \; i = 1, 2, \ldots, N \tag{9}$$

Then we have

$$\sum_{i=1}^{N} (p_i - \bar{p}_i)\, q_1 = G\,(q_1, \ldots, q_N) - C(q_1, \ldots, q_N) = F \tag{10}$$

which obviously means that $(p_i - p_i)$ is that part of price which may be thought of as covering fixed cost F.

In actual situations, there may be problems in allocating fixed costs among various commodities produced by a multi-product firm. For example, when actual demands deviate from the expected demands for various commodities, short-run technology, which is optimal for the expected demand, may not necessarily be optimal with respect to actual or realised demand in the long-run. In this case the long-run cost function is irrelevant for the allocation of fixed cost in the short- run. Therefore, fixed costs must be allocated directly from the short- run cost function. However, the prices computed using normally used arbitrary methods based on relative outputs, gross revenue or attributable cost[2] for allocating fixed joint costs among various commodities violate some or all of the five axioms described above. Mirman, Samet and Tauman (1983) have proposed amendments in additivity axiom (Axiom 5) with the addition of Axiom 6 to show that there exist modified A-S prices for a short-run cost function that satisfy this new set of axioms.

Axiom 5* : If the short-run cost function of a multi-product firm is given by

$$G = C(q_1, \dots, q_N) + F$$

and C is decomposed into $C_1, C_2, \dots, C_M$

2
$$p_i = \frac{C_i}{q_i} + \frac{f_i\ F}{q_i}, \quad i = 1, 2, \dots, N$$

$$\text{where } f_i = \frac{q_i}{\Sigma q_i} \quad \text{(Relative output method)}$$

$$= \frac{p_i\ q_i}{\Sigma\ p_i\ q_i} \quad \text{(Relative revenue method)}$$

$$= \frac{C_i}{\Sigma C_i} \quad \text{(Attributable cost method).}$$

such that $\sum_{j=1}^{M} C_j = C$, then it is possible to decompose

F into $F_1, F_2, \ldots, F_M$ such that $\sum_{j=1}^{M} F_j = F$ and

$$p\,(C + F) = \sum_{j=1}^{M} P(C_j + F_j)$$

where p is a N x 1 vector of prices.

Axiom 6 : The part F_i of fixed cost F that is associated with component C_i of variable cost C should be at least as large as F_j whenever the part C_j of C is at least as large as C_i. That means

$$C_i \geq C_j \text{ implies } F_i > F_j$$

There now exists a price mechanism that satisfies axioms 1, 2, 3, 4, 5* and 6 which is given as

$$p_i = (1 + \frac{F}{C} \quad p_i^*, \; i = 1, 2, \ldots, N \qquad (11)$$

$$\text{where } p_i^* = \int_0^1 \frac{\delta C(tq_1, tq_2, \ldots, tq_N}{\delta q_i} \, dt$$

are the A-S prices associated with variable cost. We can see immediately that the modified A-S prices in (11) are the prices derived by distributing fixed cost in proportion to allocable variable cost among different commodities of a multi-product firm. We can alternatively write (11) as

$$p_i = \frac{p_i^* \, q_i}{q_i} + \frac{f_i \; F}{q_i} \quad i = 1, 2, \ldots, N \qquad (12)$$

$$\text{where } f_i = \frac{p_i^* \; q_i}{\sum_{i=1}^{N} p_i^* q_i}$$

Hence for a multi-product public utility having cost function satisfying axioms 1, 2, 3, 4, 5* and 6, the prices derived by attributable cost method are subsidy-free.

4. Cross-subsidisation with Balanced Budget for a Welfare Maximising Public Utility

We have observed in Sections 2 and 3 of Ch. 6 that A-S prices or cross-subsidy-free prices for a public utility are supply determined or prices calculated taking into account information about cost structure of a multi-product firm. However, prices determined by a welfare maximising firm with a balanced budget with either efficiency or equity objective may not be cross-subsidy-free. The familiar Ramsey price mechanism suggests that price-cost mark-up for a commodity supplied by a publicly regulated firm should be inversely proportional to its own price elasticity of demand. Assuming constant returns to scale in production and interdependent demands for a multi-product firm (take for example a firm producing two commodities), Ramsey price mechanism is described by the following formulae (if the firm has both efficiency and equity objectives).[3]

$$\frac{p_1 - m_1}{p_1} = \frac{e_{22}(\bar{b}R_1 - 1) - e_{12}(\bar{b}R_2 - 1)}{e_{11}e_{22} - e_{12}e_{21}} \tag{13}$$

$$\frac{p_2 - m_2}{p_2} = \frac{e_{11}(\bar{b}R_2 - 1) - e_{21}(\bar{b}R_1 - 1)}{e_{11}e_{22} - e_{12}e_{21}} \tag{14}$$

where p_i : price of i-th commodity

m_i : constant marginal cost of i-th commodity

e_{ij} : elasticity of demand for i-th commodity with respect to price of j-th commodity

3. See Feldstein (1972) and Murty (1987)

$$b = \sum_{h=1}^{H} b_h/H$$ average of income distributional

weights where b_h is the income distributional weight assigned to the h-th individual, (h = 1, 2 . . . H)

$$R_i = \frac{\sum_{h=1}^{H} q_i^h \, b_h}{q_i \, b}$$ distributional characteristic of

i-th commodity where q_i^h is consumption of i-th commodity by h-th individual and

$$q_i = \sum_{h=1}^{H} q_i^h$$

If demands are independent (cross-price effects are zero) we have

$$\frac{p_1 - m_1}{p_1} = \frac{(\bar{b}R_1 - 1)}{e_{11}} = \frac{(1 - \bar{b}R_1)}{|e_{11}|} \qquad (13')$$

$$\frac{p_2 - m_2}{p_2} = \frac{(\bar{b}R_2 - 1)}{e_{22}} = \frac{(1 - \bar{b}R_2)}{|e_{22}|} \qquad (14')$$

The Ramsey price mechanism described in equations (13') and (14') clearly brings out the trade-off between equity and efficiency objectives in determining the welfare maximising prices for a publicly regulated multi-product firm. If a commodity is distributionally more important (having higher R_i) its price will be relatively lower, given the demand elasticity. However, a commodity which is distributionally more important may be having lower elasticity of demand because it is a necessity. The lower demand elasticity means higher price cost mark-up for that commodity as implied by formulae (13') and (14').

The assumption of constant returns to scale may not be tenable in the case of public utilities like Postal Services, Electricity Supply, etc. As explained in Sections 2 and 3, there may be increasing returns to scale for a multi-product firm supplying postal services. In this general case, Ramsey price mechanism may be described by the following equations[4]

$$\frac{m_1}{p_1} = \frac{e_{s1}}{e_{s1}+1} = \frac{1-\bar{b}R_1}{e_{d1}} \tag{15}$$

$$\frac{m_2}{p_2} = \frac{e_{s2}}{e_{s2}+1} = 1\ \frac{1-\bar{b}R_2}{e_{d2}} \tag{16}$$

where e_{si} : Own price elasticity of supply of i-th commodity

e_{di} : own price elasticity of demand of i-th commodity

$i = 1, 2.$

It is now clear that in the case of a budget-balancing multi-product firm with equity and efficiency objectives, the optimal prices are not cross-subsidy-free. Some commodities have to be fixed prices higher than their marginal costs for giving subsidies to other commodities. Alternatively we may consider a case in which a public utility has access to the revenue raised through commodity taxes, income taxes, etc., to finance its production. The second best prices of a public utility will then be determined after taking into account the social cost of raising revenue through pricing of its services, commodity taxes, income taxes, etc.[5]

5. Aumann-Shapley Prices for a Multi-Product Public Utility and Price Subsidies from General Revenue

We have shown in Sections 2 and 3 of Ch. 6 that cross-subsidisation by a public utility results in Pareto inefficient

4. See Jha and Murty (1987).
5. See Murty (1983).

prices and cross subsidisation is inevitable for a welfare maximising utility with efficiency and equity objectives. The problem then is to examine whether it is possible to have a case in which a public utility has Pareto efficient prices and welfare maximisation with equity and efficiency objectives is achieved through general revenue policies of government. In the literature of optimal commodity taxation we know that with the assumption of constant returns to scale in private production, there are constant producer prices which are Pareto efficient and there exist second best consumer prices/taxes for a welfare maximising government with equity and efficiency objectives that meet a pre-specified government revenue requirement. But if we have utility services in the economy, with increasing returns to scale in their production, public regulation may be necessary to provide these services at Pareto efficient/first best prices. Assuming that a publicly regulated utility is guided solely by Pareto efficient prices and the private sector production takes place with constant returns to scale, welfare maximising taxes/subsidies on public utility services and private sector commodities can be determined subject to a government revenue constraint.

Let there be n private sector commodities and m public utility services in the economy and constant returns to scale in private production and increasing returns to scale in the production of the services provided by a publicly regulated joint product firm so that it has Pareto efficient prices for its services, A-S prices which are defined for given levels of supply of these services. On the other hand, private sector commodities have constant producer prices which are equal to their marginal costs.[6] The consumer prices are now defined for the private sector commodities as

$$p_i = d_i + t_i \qquad i = 1, \ldots, N \qquad (17)$$

6. Marginal costs for private sector commodities are constant because of the assumption of constant returns in production.

where p_i, d_i and t_i respectively represent consumer prices, product prices and taxes.

In the case of the public utility which supplies goods j, j = N + 1 , . . . , N + M their producer prices are already fixed by consideration of fully distributed cost, e.g., A-S prices. Let d_j, j = N + 1 , . . . , N + M be the levels of these prices. As we have seen in Sections 2 and 3 these meet the requirement that each d_j corresponds to a level of supply of commodity/service j. If d_j were to change so would the supply of commodity j and *vice versa*. It is also clear that we have assumed that demand is always forthcoming to meet the supply implied by A-S price d_j, j = N + 1 , . . . , N + M.

The government is, however, free to price these commodities in a different manner for the consumers. We can assume that these commodities/services are procured by the government at prices d_j which is then free to tax/subsidise various constituents of the public utility's service. The assumption that demand will be forthcoming to meet the supply is again implicit. Thus final consumer prices are

$$p_j = d_j + t_j \quad j = N + 1 , \ldots , N + M \tag{17}$$

where p_j are the consumer prices and t_j are the tax/subsidy rates. The overall revenue constraint of the government is now defined as :

$$\sum_{i=1}^{N+M} t_i q_i = R \tag{18}$$

where q_i, i = 1 , . . . , N, N + 1 , . . . , N + M are the quantities of commodities demanded in the economy and R is the exogenously given revenue requirement of the government. The welfare function of the government is now defined as

$$W (V_1 , V_2 , \ldots , V_H) \tag{19}$$

where $V_h = V_h (P_1 , \ldots , P_N, P_{n+1} , \ldots , P_{N+M}, I_h)$ is the indirect utility function of the h-th individual and I_h is his income.

Maximisation of (19) subject to (18) with respect to t_i yields welfare maximising t_i for the given level of production of public utility services. Hence in this approach once cross-subsidy-free prices for the public utility are determined, the government evaluates these public utility services along with other goods in the economy. The second best problem solved at this stage gives us welfare maximising tax/subsidy rates for private sector commodities and public utilities services which meet the specific equity and efficiency objectives of the government while, at the same time, meeting the revenue requirements of the government.

7.6 Conclusions

We have discussed alternative pricing schemes (first best or second best) for a multi-product public utility with joint costs and increasing returns to scale. The first best prices are cross-subsidy-free or A-S prices. The A-S prices can be computed given the long run cost functions of a multi-product public utility. However, in the case of short-run cost functions with a distinction between fixed and variable costs, the A-S prices can be computed only for a class of cost functions. The cross-subsidy-free prices in this case correspond to prices computed with the familiar attributable cost method for a multi-product public utility.

The second best or Ramsey prices are not cross-subsidy-free. For a welfare maximising public utility with balanced budget, these prices are inevitable. Given the alternative sources of revenue like commodity taxes and income taxes to finance the public utility production, the welfare maximising prices for utility services with the assumption of balanced budget for public utility may not be globally optimal. For fixing globally optimal public utility prices we have to consider the social cost of raising revenue through prices of public utility services, and other public sector commodities and income taxes, commodity taxes, etc. to fund public utility production.

We have suggested a pricing scheme for public utility services that takes A-S prices as the first best producer prices and subsidised or tax inclusive prices as consumer prices. In an economy having a public utility, private sector production, increasing returns to scale for public utility and constant returns to scale in private production, A-S prices for the public utility and prices that are equal to marginal costs (constant) for private sector commodities are Pareto optimal prices for given level of production of public utility services. Given these prices for public utility services and private sector commodities, the second best prices *á la* Ramsey can be determined by the welfare maximising government with revenue, efficiency and equity objectives.