CAN THE PROGRESSIVE REVENUE SHARING CRITERIA LEAD TO REGRESSIVE RESPONSES? AN EMPIRICAL QUESTION

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No. 2   April 1995

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Abstract

This paper analyses the responses of the three allocative criteria for federal fiscal transfer to States, namely, the distance criterion, the inverse-income criterion and the integrated criterion to the changes in per capita income and/or population size of a participating State. It derives necessary and sufficient conditions for an allocative criterion to result in progressive responses. The conditions are complex and not amenable to an unambiguous interpretation. Relative merits of these criteria in terms of conditions for regressive responses are also studied. It is found that the integrated criterion is least prone and the inverse-income criterion most prone to regressive responses, at least among the high income States.

CAN THE PROGRESSIVE REVENUE SHARING CRITERIA LEAD TO REGRESSIVE RESPONSES? AN EMPIRICAL QUESTION

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I.   Introduction

In federal fiscal systems, a national government devolves a part of its revenue among the sub-national governments to correct for vertical fiscal imbalance between the different levels of governments. The allocation of revenue devolution among the participating sub-national (State) governments aims at correcting for the horizontal fiscal imbalance among the States. For attaining this end, alternative allocative criteria have been adopted in different federations. The achievement of an allocation criterion is judged in terms of progressivity of the devolution with reference to income of States, since a poorer State, generally, has the lower fiscal capacity.   Recently, the characteristics of three allocative criteria, namely, the distance criterion, the inverse-income criterion and integrated criterion have been analysed. Srivastava and Aggarwal (1993) gives an analysis of the first two criteria while the third criterion is proposed and its merits vis-a-vis the other two criteria are studied in Aggarwal and Srivastava (1994).   All the three allocative criteria are found to be progressive with respect to per capita income (hereinafter referred to as income) of States in the sense that per capita share of a State in revenue devolution decreases with an increase in its income, ceteris paribus. On the other hand, the share of a State, with all the three allocative criteria, is found to decline following an increase in its population size, other things remaining the same. Among the three allocative criteria, the integrated criterion that is most progressive with respect to income is also found to be most

responsive to population size of States. Under certain pattern of changes in income and population, the size effect may dominate the income effect resulting in regressive changes in the allocation of revenue devolution i.e., a reduction in the progressivity of an allocative criterion. It is in this context that this paper attempts to identify patterns of changes in income and population of States which may lead to a reduction in the progressivity of an allocative criterion, if any.

Section II of this paper gives salient features of the three allocative criteria. Section III analyses, if the changes in income (y_i) and population (N_i) of a State can, in effect, result in a decline in the progressivity of any of the three allocative criteria, discussed above. Section IV contains concluding remarks.

It has been assumed throughout that y_i and N_i are independently distributed. The mechanism that generates y_i and N_i is exogenous to individual criteria and the shares determined by them. The results are independent of the mechanism that generates y_i and N_i. The different criteria respond differently when the same changes in y_i and N_i are fed into them. The difference is due to the internal structure of the criterion, and the purpose of the paper is to bring this out.

II.   Allocation Criteria: Salient Features

a.   Description of Allocation Criteria

The three allocative criteria referred to above can be described by a general formula given by

$$s_i = N_i R_i / \Sigma_i (N_i R_i) \qquad\qquad i = 1,2,3,\ldots n \qquad\qquad (1)$$

Where s_i denotes share of the ith State in revenue devolution and R_i denotes relative deficiency in fiscal capacity of the ith State. Population size `N_i' of the ith State is interpreted as

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weight assigned to its relative deficiency in fiscal capacity `R_i'. The specific forms of R_i give rise to different allocative criteria. The shares of the ith State a_i, b_i and c_i under the distance, the inverse-income and the integrated criteria are obtained by substituting R_i in equation (1) by $y_n - y_i$, $(1/y_i)$ and $((y_n - y_i)/y_i)$ respectively. The shares a_i, b_i and c_i can be written as:

$$a_i = \frac{N_i(y_n - y_i)}{\Sigma_i(N_i(y_n - y_i))} = \alpha N_i(y_n - y_i) \tag{2}$$

$$b_i = \frac{N_i/y_i}{\Sigma_i(N_i/y_i)} = \beta N_i/y_i \tag{3}$$

$$c_i = \frac{N_i(y_n - y_i)/y_i}{\Sigma_i(N_i(y_n - y_i)/y_i)} = \Gamma N_i(y_n - y_i)/y_i \tag{4}$$

where

$$\alpha = 1/\Sigma_i(N_i(y_n - y_i)) \tag{5}$$

$$\beta = 1/\Sigma_i(N_i/y_i) \tag{6}$$

$$\Gamma = 1/\Sigma_i(N_i(y_n - y_i)/y_i) \tag{7}$$

It has been found that income and/or size responsiveness of State shares a_i, b_i and c_i is not amenable to unambiguous interpretation as to the progressivity of these allocative criteria. On the other hand, an unambiguous interpretation is possible when the analysis is cast in terms of responsiveness of per capita State shares a_i^*, b_i^* and c_i^* (see Aggarwal and Srivastava, 1994 and Srivastava and Aggarwal, 1993). The per capita shares under the three allocative criteria are given by

$$a_i^* = \frac{a_i}{N_i} = \frac{y_n - y_i}{\Sigma(N_i(y_n - y_i))} = \alpha(y_n - y_i) \tag{8}$$

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$$b_i{}^* = \frac{b_i}{N_i} = \frac{1/y_i}{\Sigma(N_i/y_i)} = \beta/y_i \qquad (9)$$

$$c_i{}^* = \frac{c_i}{N_i} = \frac{(y_n - y_i)/y_i}{\Sigma(N_i(y_n - y_i)/y_i)} = \Gamma(y_n - y_i)/y_i \qquad (10)$$

b.   Responsiveness of per capita shares

All the three allocative criteria are progressive with respect to income in the sense that per capita shares of a State decline with rise in its income, other things remaining the same. This is testified by the partial differentials of per capita shares of States with respect to income, which are given by

$$\frac{\partial a_i{}^*}{\partial y_i} = - \frac{a_i(1 - a_i)}{N_i(y_n - y_i)} < 0 \qquad (11)$$

$$\frac{\partial b_i{}^*}{\partial y_i} = - \frac{b_i(1 - b_i)}{N_i\, y_i} < 0 \qquad (12)$$

$$\frac{\partial c_i{}^*}{\partial y_i} = - \frac{y_n\, c_i(1 - c_i)}{N_i\, y_i(y_n - y_i)} < 0 \qquad (13)$$

Among the three allocative criteria, the integrated criterion is most progressive with respect to income of States, other things remaining the same (see Aggarwal and Srivastava, 1994).

All the three allocative criteria are regressive with respect to population in the sense that per capita shares of a State decline with rise in its population. This is evident from the following partial differentials of per capita shares with respect to size of States:

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$$\frac{\partial a_i^*}{\partial N_i} = - a_i^{*2} < 0 \qquad (14)$$

$$\frac{\partial b_i^*}{\partial N_i} = - b_i^{*2} < 0 \qquad (15)$$

$$\frac{\partial c_i^*}{\partial N_i} = - c_i^{*2} < 0 \qquad (16)$$

For all the three allocative criteria the larger the initial share of a State, the larger would be the decline in its share following an increase in its population. The integrated criterion that is most progressive with respect to income also leads to a greater decline in the progressivity following population growth (see Aggarwal and Srivastava, 1994).

III. Conditions for Regressive Responses: Net Impact of Income and Size Effects

The net impact of income and population changes of a State on its share, with all the three allocative criteria, can be analysed in terms of total differentials of per capita shares as:

$$da_i^* = \frac{\partial a_i^*}{\partial N_i} \, dN_i + \frac{\partial a_i^*}{\partial y_i} \, dy_i \qquad (17)$$

$$db_i^* = \frac{\partial b_i^*}{\partial N_i} \, dN_i + \frac{\partial b_i^*}{\partial y_i} \, dy_i \qquad (18)$$

$$dc_i^* = \frac{\partial c_i^*}{\partial N_i} \, dN_i + \frac{\partial c_i^*}{\partial y_i} \, dy_i \qquad (19)$$

Alternatively equations (17) to (19) can be expressed as:

$$da_i^* = - a_i^{*2} \, dN_i - \frac{1}{y_n - y_i} \, a_i^*(1 - a_i) dy_i \qquad (20)$$

$$db_i^* = - b_i^{*2} \, dN_i - \frac{1}{Y_i} \, b_i^*(1 - b_i) dy_i \tag{21}$$

$$dc_i^* = - c_i^{*2} \, dN_i - \frac{Y_n}{Y_i(Y_n - Y_i)} \, c_i^*(1 - c_i) dy_i \tag{22}$$

These equations imply that

$$da_i^* < 0 \quad \text{if} \quad dN_i > - z_i \, dy_i \tag{23}$$

$$db_i^* < 0 \quad \text{if} \quad dN_i > - z_i^* \, dy_i \tag{24}$$

$$\text{and} \quad dc_i^* < 0 \quad \text{if} \quad dN_i > - z_i^{**} \, dy_i \tag{25}$$

where

$$z_i = \frac{1}{Y_n - Y_i} \, \frac{1-a_i}{a_i^*} \quad > 0 \tag{26}$$

$$z_i^* = \frac{1}{Y_i} \, \frac{1-b_i}{b_i^*} \quad > 0 \tag{27}$$

$$z_i^{**} = \frac{Y_n}{Y_i(Y_n-Y_i)} \, \frac{1-c_i}{c_i^*} \quad > 0 \tag{28}$$

It is evident from these results that responses of all the three allocative criteria are necessarily progressive if both dN_i and dy_i are of the same sign, i.e., both increase or decrease simultaneously, provided a_i^*, b_i^* and c_i^* are not homogenous of degree zero in y_i and N_i. Further, regressive responses will be observed when an increase (decrease) in the population of the ith State exceeds z_i times the decline (rise) in its income in the case of the distance criterion, z_i^* times the decline (rise) in its income in the case of the inverse-income criterion, and z_i^{**} times the decline (rise) in its income in the case of the integrated criterion *ceteris paribus* as is shown in the Figure.

In the Figure, dy_i and dN_i corresponding to the ith State are represented on the x-axis and y-axis respectively. The figure depicts the situation when $Z_i^* < Z_i < Z_i^{**}$, that is true at least for the high income States as will be discussed later. The lines AA', BB' and CC' correspond to $dN_i = -Z_i^* dy_i$, $dN_i = -Z_i dy_i$ and $dN_i = -Z_i^{**} dy_i$ respectively. The zones represented by angles H, I and J relate to regressive responses and the zones represented by angles (D+E) and (F+G) show the region of progressive responses. For the inverse-income, the distance and the integrated criteria, the regions of regressive response are represented by the angles (H+I+J), (I+J) and J respectively. To illustrate, in the case of inverse-income criterion, any point in the region (H+I+J) for $dN_i < 0$ and $dy_i > 0$, i.e., in the fourth quadrant will correspond to the condition $dN_i < -Z_i^* dy_i$ implying $db_i^* > 0$ and hence regressive response. Similarly, any point in the region (H+I+J) for $dN_i > 0$ and $dy_i < 0$, i.e., in the second quadrant satisfies the condition $dN_i > -Z_i^* dy_i$ implying $db_i^* < 0$ and hence regressive response.

From equations (23) to (28), it may be noted that with all the allocative criteria, the conditions leading to progressive or regressive responses to changes in income and population, depend on the initial income levels and shares of the States. These conditions are complex and not amenable to unambiguous interpretation[1]. Thus, the question, whether or not an allocative criterion would lead to progressive changes in the allocation of devolution following a simultaneous change in, income and population of a State, has to be resolved empirically by looking into initial per capita shares of States for and testing whether the resultant allocation of devolution after the change is more progressive.
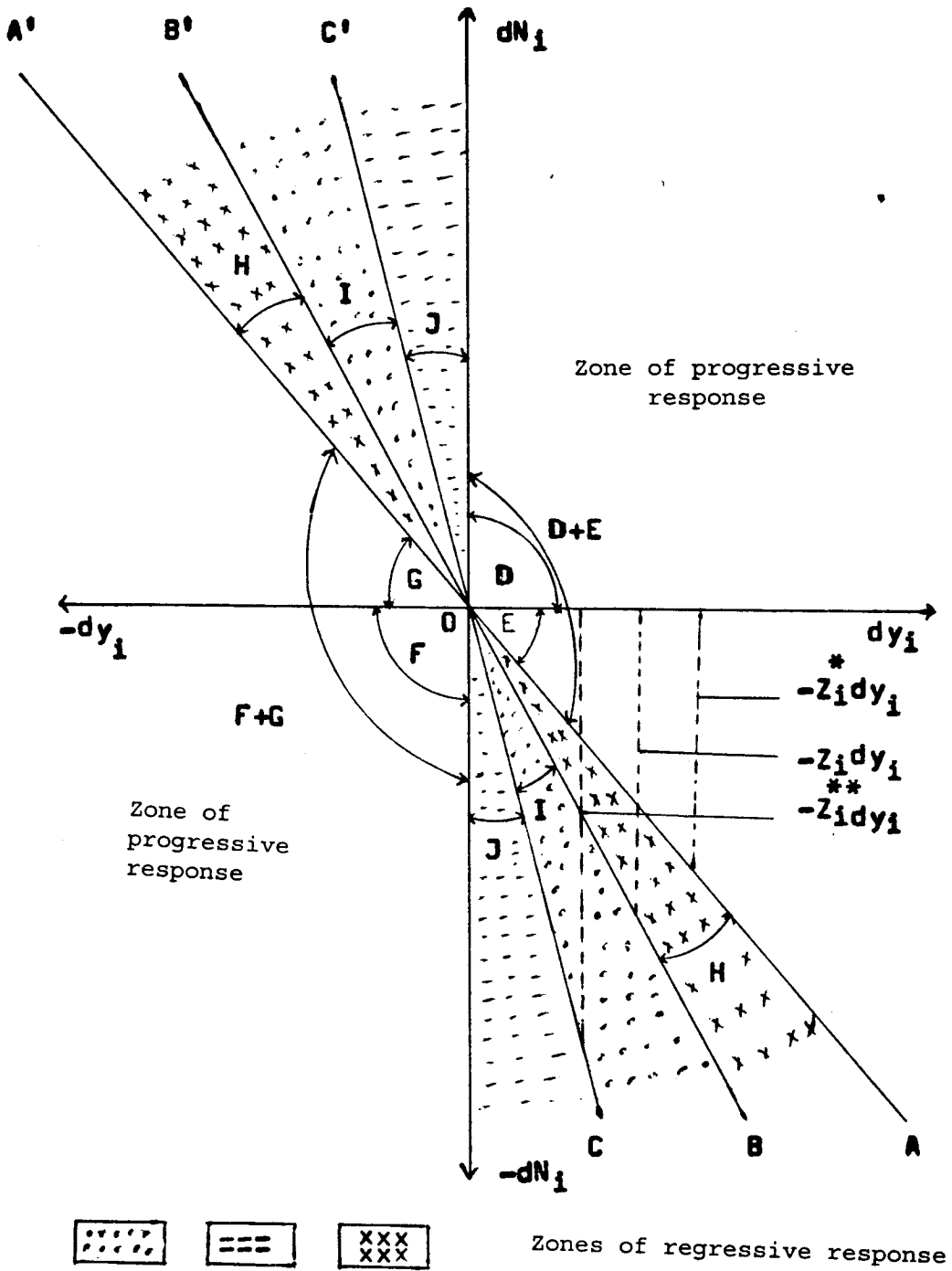
Figure

Something can be said about the relative merits of the three allocative criteria in terms of stronger or weaker conditions to be met for an allocative criterion to result in regressive changes in the allocation of devolution. For this purpose, let us consider the following ratios:

$$\frac{z_i^{**}}{z_i} = \frac{y_n}{y_i} \frac{a_i^*}{c_i^*} \frac{1 - c_i}{1 - a_i}$$

or $$\frac{z_i^{**}}{z_i} = \frac{y_n}{y_i} \frac{a_i}{c_i} \frac{1 - c_i}{1 - a_i}$$

or $$\frac{z_i^{**}}{z_i} = \frac{y_n}{y_i} \frac{(1/c_i - 1)}{(1/a_i - 1)} \tag{29}$$

Similarly,

$$\frac{z_i^{**}}{z_i^*} = \frac{y_n}{y_n - y_i} \frac{(1/c_i - 1)}{(1/b_i - 1)} , \text{ and} \tag{30}$$

$$\frac{z_i}{z_i^*} = \frac{y_i}{y_n - y_i} \frac{(1/a_i - 1)}{(1/b_i - 1)} \tag{31}$$

Equation (29) indicates that at least among the high income (with $Y_i > Y_n/2$) States, for $a_i > c_i$, z_i^{**} will be greater than z_i. Similarly, equation (30) indicates that at least among the high income States, for $b_i > c_i$, z_i^{**} will be greater than z_i^*. Further, equation (31) indicates that at least among the high income States, for $b_i > a_i$, z_i will be greater than z_i^*. These results imply that at least among the high income States (for $b_i > a_i > c_i$), $z_i^{**} > z_i > z_i^*$. This means that a stronger negative relationship between the changes in population size and per capita income of States would be required for the integrated criterion to result in regressive responses at least among the high income States. This can be interpreted as a merit of the integrated criterion over the other two criteria. On the other hand, even a

weaker negative relationship between the changes in population size and per capita income of States would drive the inverse-income criterion to lead to regressive responses at least among the high income States. Thus, among the three allocative criteria, the integrated criterion is least prone and the inverse-income criterion most prone to regressive responses in allocation of revenue devolution, at least among the high income States.

IV.   Concluding Remarks

This paper analyses the responses of the three allocative criteria, namely, the distance criterion, the inverse-income criterion and the integrated criterion to the changes in the distribution of income (y_i, N_i) among the participating States. A sufficient condition for an allocative criterion to result in a progressive response to a change in the pattern of distribution (y_i, N_i) is that the rise in per capita income of a State is accompanied by an increase in the size of the State. The necessary and sufficient conditions for the same are not amenable to unambiguous interpretation analytically. However, these conditions provide a framework in which whether or not a criterion would result in progressive responses could be resolved empirically. Also, something can be said about the relative merits of these criteria in terms of conditions for regressive response to changes in the distribution. The integrated criterion is least prone and the inverse-income criterion most prone to regressive responses in the allocation of revenue devolution, at least among the high income States.

* The authors wish to acknowledge the useful remarks of an anonymous referee and the adept secretarial assistance provided by Ms. Promila Rajvanshi.

1. The terms Z_i, Z_i^* and Z_i^{**} can be expressed as follows:

$$Z_i = N_i \left(\frac{1}{y_n - y_i}\right) \left(\frac{1 - a_i}{a_i}\right)$$

$$Z_i^* = N_i \left(\frac{1}{y_i}\right) \left(\frac{1 - b_i}{b_i}\right)$$

$$Z_i^{**} = N_i \left(\frac{y_n}{y_i(y_n - y_i)}\right) \left(\frac{1 - C_i}{C_i}\right)$$

In the distance criterion, the term $(1 - a_i)$ represents share of all 'other' States. Thus, $((1 - a_i)/a_i)$ is the share of remaining States relative to a given State. Similarly, $((1 - b_i)/b_i))$ and $((1 - C_i)/C_i)$ represent the relative shares of all other States under the inverse-income and the integrated criteria respectively. The terms $(1/(y_n - y_i))$, $(1/y_i)$ and $(y_n/y_i(y_n - y_i))$ represent change in deficiency relative to original deficiency of a State in the distance, the inverse-income and the integrated criteria respectively. These terms emanate from the term $(\delta W_i/\delta y_i)/W_i$ where W_i's are the weights indicating relative deficiency of a State in each criterion. Thus, for the distance criterion,

$$W_i = y_n - y_i, \quad \delta W/\delta y_i = -1,$$

and $(\delta W_i/\delta y_i)/W_i = -1/(y_n - y_i)$

For the inverse-income cirterion,

$$W_i = 1/y_i, \quad \delta W_i/\delta y_i = -1/y_i^2,$$

and $(\delta W_i/\delta y_i)/W_i = -1/y_i$

For the integrated criterion,

$$W_i = (Y_n - Y_i)/Y_i = (Y_n/Y_i - 1),$$
$$\partial W_i/\partial Y_i = -Y_n/Y_i^2,$$

and $(\partial W_i/\partial Y_i)/W_i = -Y_n/(Y_i(Y_n - Y_i))$

Thus, $-Z_i$, $-Z_i^*$ and $-Z_i^{**}$ can be writeen as

$$
\begin{bmatrix} \text{Critical} \\ \text{benchmark} \\ \text{value} \end{bmatrix}
=
\begin{bmatrix} \text{Original} \\ \text{population} \\ \text{of a State} \end{bmatrix}
\times
\begin{bmatrix} \text{Change in} \\ \text{deficiency} \\ \text{relative} \\ \text{to original} \\ \text{deficiency} \end{bmatrix}
\times
\begin{bmatrix} \text{Share of} \\ \text{other} \\ \text{States} \\ \text{relative} \\ \text{to its} \\ \text{own share} \end{bmatrix}
$$

This formulation of the critical values of `Z' is not amenable to an unambiguous interpretatioin.

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